Supplement to “Practical Kolmogorov-Smirnov Testing by Minimum Distance Applied to Measure Top Income Shares in Korea”

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Abstract

We provide proofs of the results and data descriptions for the empirical work reported in “Practicable Kolmogorov-Smirnov Testing and Top Income Shares in Korea” by Cho, Park, and Phillips (2016).

1 Introduction

In the technical supplement that follows in Section 2 we provide proofs of the results stated Cho, Park, and Phillips (2016). Section 3 provides a data description for the empirical application to Korean income data reported in the paper. Section 4 provides some supplementary information about the hypotheses under test.

2 Technical Supplement

Proof of Theorem 1: For each j we have $\widehat{p}_{n,j} \stackrel{\text{a.s.}}{\to} p_j$, so that the uniform strong law $\sup_{\theta\in\Theta} | \sum_{j=1}^{k}\{F_j(\theta)-\widehat{p}_{n,j}\}^2 - \sum_{j=1}^{k}\{F_j(\theta)-p_j\}^2| \stackrel{\text{a.s.}}{\to} 0$ holds, which implies that $\widehat{\theta}_n \stackrel{\text{a.s.}}{\to} \arg\min_{\theta\in\Theta}\sum_{j=1}^{k}\{F_j(\theta)-p_j\}^2$. By Assumption 1($iii$). θ_o is unique in Θ. Therefore, $\widehat{\theta}_n \stackrel{\text{a.s.}}{\to} \theta_o$ as desired. ■

Proof of Theorem 2: By Taylor expansion: $F_j(\theta) = F_j(\theta_o) + \nabla_\theta' F_j(\theta_o)(\theta-\theta_o) + O((\theta-\theta_o)^2)$, so that $Q_n(\theta) = \sum_{j=1}^{k}\{\widehat{y}_j - Z_j'(\theta-\theta_o)[1+O(\theta-\theta_o)]\}^2$. Therefore, $(\widehat{\theta}_n-\theta_o)[1+O(\widehat{\theta}_n-\theta_o)] = (Z'Z)^{-1}Z'\widehat{Y}$ using least squares, which in turn implies

$$(\widehat{\theta}_n-\theta_o)[1+O_{\mathbb{P}}(\widehat{\theta}_n-\theta_o)] - (Z'Z)^{-1}Z'Y = (Z'Z)^{-1}Z'(\widehat{Y}-Y).$$

Note that $\sqrt{n}(\widehat{Y}-Y) \Rightarrow W$ and $Z'Y=0$ from the first-order condition for θ_o, implying that

$$\sqrt{n}(\widehat{\theta}_n-\theta_o) = \sqrt{n}(Z'Z)^{-1}Z'(\widehat{Y}-Y) + O_{\mathbb{P}}(n^{-1/2}) \Rightarrow (Z'Z)^{-1}Z'W, \tag{1}$$

completing the proof. ■

Proof of Theorem 3: (i) Since, for each $j=1,\ldots,k$, $F_j(\widehat{\theta}_n) = F_j(\theta_*) + \nabla_\theta' F_j(\theta_*)(\widehat{\theta}_n-\theta_*) + O_{\mathbb{P}}(n^{-1})$, it follows that

$$\begin{aligned}\sqrt{n}(\widehat{p}_{n,j}-F_j(\widehat{\theta}_n)) &= \sqrt{n}(\widehat{p}_{n,j}-F_j(\theta_*)) - \nabla_\theta' F_j(\theta_*)\sqrt{n}(\widehat{\theta}_n-\theta_*) + O_{\mathbb{P}}(n^{-1/2})\\ &= \sqrt{n}(\widehat{p}_{n,j}-F_j(\theta_*)) - Z_j'(Z'^{-1}Z'\sqrt{n}(\widehat{Y}-Y) + O_{\mathbb{P}}(n^{-1/2})\\ &\Rightarrow \mathcal{B}^o(p_j) - Z_j'(Z'Z)^{-1}Z'W,\end{aligned}$$

where the last equality and the weak convergence follows from (1). The result holds for every j and jointly, so it follows that $\widehat{S}_n \Rightarrow W - Z(Z'Z)^{-1}Z'W = MW = G$. Therefore, continuous mapping delivers the limit result for the test statistic $\widehat{T}_n \Rightarrow \max[|g_1|, \ldots, |g_k|]$.

(*ii*) Since for each $j = 1, \ldots, k$, $F_j(\widehat{\theta}_n) = F_j(\theta_o) + \nabla'_\theta F_j(\theta_o)(\widehat{\theta}_n - \theta_o) + O_{\mathbb{P}}(n^{-1})$, it follows that

$$\widehat{p}_{n,j} - F_j(\widehat{\theta}_n) - (p_j - F_j(\theta_o)) = \widehat{p}_{n,j} - p_j - \nabla'_\theta F_j(\theta_o)(\widehat{\theta}_n - \theta_o) + O_{\mathbb{P}}(n^{-1}). \tag{2}$$

Given the definition of $y_j := p_j - F_j(\theta_o)$, it follows that

$$\begin{aligned}\sqrt{n}\{\widehat{p}_{n,j} - F_j(\widehat{\theta}_n) - y_j\} &= \sqrt{n}(\widehat{p}_{n,j} - p_j) - \nabla'_\theta F_j(\theta_o)\sqrt{n}(\widehat{\theta}_n - \theta_o) + O_{\mathbb{P}}(n^{-1/2}) \\ &= \sqrt{n}(\widehat{p}_{n,j} - p_j) - Z'_j(Z'Z)^{-1}Z'\sqrt{n}(\widehat{Y} - Y) + O_{\mathbb{P}}(n^{-1/2}) \\ &\Rightarrow \mathcal{B}^o(p_j) - Z'_j(Z'Z)^{-1}Z'W,\end{aligned}$$

where the last equality and the weak convergence hold by (1). The result holds for all j and jointly, so that $\sqrt{n}(\widehat{S}_n - Y) \Rightarrow G$, and the desired result follows.

(*iii*) As in the proof of Theorem 1, we note that, for each j, $F_j(\widehat{\theta}_n) = F_j(\theta_o) + Z'_j(\widehat{\theta}_n - \theta_o) + o_{\mathbb{P}}(n^{-1/2})$, implying that

$$\begin{aligned}\widehat{p}_{n,j} - F_j(\widehat{\theta}_n) &= \widehat{p}_{n,j} - p_j + p_j - F_j(\theta_o) - Z'_j(\widehat{\theta}_n - \theta_o) + o_{\mathbb{P}}(n^{-1/2}) \\ &= \widehat{p}_{n,j} - p_j + y_j - Z'_j(Z'Z)^{-1}Z'Y - Z'_j(Z'Z)^{-1}Z'(\widehat{Y} - Y) + o_{\mathbb{P}}(n^{-1/2}) \\ &= \widehat{p}_{n,j} - p_j + n^{-1/2}\xi_j - Z'_j(Z'Z)^{-1}Z'(\widehat{Y} - Y) + o_{\mathbb{P}}(n^{-1/2}),\end{aligned}$$

where the second equality holds by the definition of $y_j := p_j - F_j(\theta_o)$ and the fact that $(\widehat{\theta}_n - \theta_o) = (Z'Z)^{-1}Z'Y + (Z'Z)^{-1}Z'(\widehat{Y} - Y) + O_{\mathbb{P}}(n^{-1})$, and the third equality holds by virtue of the local alternative $\mathcal{H}_\ell$ and the definition of $\xi_j := y_j - Z'_j(Z'Z)^{-1}Z'Y$. Therefore, $\sqrt{n}\{\widehat{p}_{n,j} - F_j(\widehat{\theta}_n)\} \Rightarrow \xi_j + g_j$, which holds for every j and jointly. Therefore, $\widehat{S}_n \Rightarrow M(H + W) \sim N(MH, M\Sigma M)$, so that the test statistic $\widehat{T}_n \Rightarrow \max[|\xi_1 + g_1|, \ldots, |\xi_k + g_k|]$, as desired. ■

Proof of Lemma 1: (*i*) We note that $\partial_j F_k(\cdot, \theta_o)$ is piecewise continuous on $[c_0, c_k]$. Therefore, for each i and j, $\int_0^1 \partial_i \bar{F}_k(x, \theta_o)\partial_j \bar{F}_k(x, \theta_o)dx$ is well defined. Similarly, for each i and j, $\partial_i \bar{F}(\cdot, \theta_o)$ and $\partial_j \bar{F}(\cdot, \theta_o)$ are continuous on $[0, 1]$ by Assumption 2(*i*), implying that $\partial_i \bar{F}(\cdot, \theta_o)\partial_j \bar{F}(\cdot, \theta_o)$ is also continuous on $[0, 1]$ and that $\int_0^1 \partial_i \bar{F}(x, \theta_o)\partial_j \bar{F}(x, \theta_o)dx$ is also well defined. Furthermore, $\partial_i \bar{F}_k(\cdot, \theta_o)\partial_j \bar{F}_k(\cdot, \theta_o)$ converges uniformly to $\partial_i \bar{F}(\cdot, \theta_o)\partial_j \bar{F}(\cdot, \theta_o)$, and $|\partial_i \bar{F}_k(\cdot, \theta_o)\partial_j \bar{F}_k(\cdot, \theta_o)|$ is uniformly bounded by $\sup_{x\in[0,1]} |\partial_i \bar{F}(x,$

$\theta_o)\partial_j\bar{F}(x,\theta_o)|$ by Theorem 2(*ii*). Thus, dominated convergence implies that

$$\int_0^1 \partial_i\bar{F}_k(x,\theta_o)\partial_j\bar{F}_k(x,\theta_o)dx \xrightarrow{k} \int_0^1 \partial_i\bar{F}(x,\theta_o)\partial_j\bar{F}(x,\theta_o)dx.$$

As this limit holds for every combination of i and j, the desired result follows.

(*ii*) By definition $\bar{\mathcal{B}}_k^o(\cdot)$ is piecewise continuous on $[0,1]$, so that for every j, $\int_0^1 \partial_j\bar{F}_k(x,\theta_o)\bar{\mathcal{B}}_k^o(x)dx$ is well defined as before. Furthermore, $\bar{\mathcal{B}}^o(\cdot)$ is continuous with probability 1, implying that $\partial_j\bar{F}(\cdot,\theta_o)\bar{\mathcal{B}}^o(\cdot)$ is also continuous on $[0,1]$ with probability 1. Therefore, for each j, $\int_0^1 \partial_j\bar{F}(x,\theta_o)\bar{\mathcal{B}}^o(x)dx$ is also well defined with probability 1. Furthermore, note that $|\partial_j\bar{F}_k(\cdot,\theta_o)\bar{\mathcal{B}}_k^o(\cdot)|$ is uniformly bounded by $\sup_{x\in[0,1]}|\partial_j\bar{F}(x,\theta_o)\bar{\mathcal{B}}^o(x)|$ with probability 1 by Assumption 2(*ii*). Therefore, dominated convergence implies that for each j,

$$\int_0^1 \partial_j\bar{F}_k(x,\theta_o)\bar{\mathcal{B}}_k^o(x)dx \xrightarrow{k} \int_0^1 \partial_j\bar{F}(x,\theta_o)\bar{\mathcal{B}}^o(x)dx,$$

with probability 1. As this result holds for every j and jointly, $U_k \overset{k}{\Rightarrow} U$. We also note that Assumptions 2(*iv* and *v*) imply that A_o^{-1} exists and $\mathbb{E}[\int_0^1\int_0^1 \nabla_\theta\bar{F}(x,\theta_o)\nabla_\theta'\bar{F}(x',\theta_o)\bar{\mathcal{B}}^o(x)\bar{\mathcal{B}}^o(x')dxdx'] = B_o$ by Fubini and Tonelli, respectively. This completes the proof. ■

Proof of Theorem 4: The desired result simply follows by combining results in Lemma 1(*i* and *ii*) using joint convergence. ■

Proof of Theorem 5: (*i*) Weak convergence as $n \to \infty$ is already provided in the proof of Theorem 3(*i*) and so we need only show weak convergence with respect to k. Note that $\bar{\mathcal{B}}_k^o(\cdot) \xrightarrow{k} \bar{\mathcal{B}}^o(\cdot)$ uniformly on $[0,1]$ with probability 1, and $\nabla_\theta\bar{F}_k(\cdot,\theta_o) \xrightarrow{k} \nabla_\theta\bar{F}(\cdot,\theta_o)$ by Assumption 2(*ii*). Finally, $A_k^{-1}U_k \overset{k}{\Rightarrow} A_o^{-1}U$ by Lemma 1. It follows that $\bar{\mathcal{B}}_k^o(\cdot)-\nabla_\theta'\bar{F}_k(\cdot,\theta_o)A_k^{-1}U_k \overset{k}{\Rightarrow} \bar{\mathcal{B}}^o(\cdot)-\nabla_\theta'\bar{F}(\cdot,\theta_o)A_o^{-1}U$ by joint convergence. Now simply apply the definitions of $\bar{\mathcal{G}}_k^o(\cdot)$ and $\bar{\mathcal{G}}^o(\cdot)$ to the left and right side of this large group weak convergence result.

(*ii*) Weak convergence as $n \to \infty$ is again provided in the proof of Theorem 3(*ii*). The proof of Theorem 5(*i*) shows that $\bar{\mathcal{G}}_k^o(\cdot) \overset{k}{\Rightarrow} \bar{\mathcal{G}}^o(\cdot)$, and $\bar{\xi}_k(\cdot) = \bar{h}_k(\cdot) \xrightarrow{k} \bar{h}(\cdot)$ uniformly on $[0,1]$ by virtue of the structure of $\bar{h}_k(\cdot)$. The desired result follows directly. ■

3 Supplementary Data Description: Korean Income Data from 2007 to 2012

Table 1 provides summary statistics of the income tax return data. We provide more detailed source and nature of the data in this subsection.

3.1 Grouped Income Data

The income tax return data are formed from several different income sources. The following items are included in the data: business income, interests and dividends (above KRW40 million), pension benefits, wage income, and other income. All these are taxable income sources. Interests and dividends (below KRW40 million), retirement income, and capital gains are not included in our data. As our main focus of here is in the distribution of high income groups, the exclusion of interest and dividend income below KRW40 million is unlikely to affect inferences. High income groups with more than KRW40 million in interest and dividend income are included in our data. Consistent with other countries, the high income groups in our data are mostly determined by wage income, business income, and interest and dividend income.

On the other hand, our data do not include all types of non-taxable income. Most non-taxable income data are not easy to verify and some income is not voluntarily reported especially that relating to financial income for high income groups. This aspect of income data produces multiple sources of measurement error and difficulties in correcting income data to include all types of non-taxable incomes. Therefore, we delimit attention to taxable income, as is commonly done for other countries, in order to maintain a consistent income definition throughout the time period studied and to minimize the effects of measurement error in the data.

3.2 Total Income Calculation

The way total income is derived from the national accounts for personal income differs depending on which of the two income tax systems is in use: negative and positive income tax systems. The negative income tax system includes almost all types of income earned by personnel and uses these to construct income tax statistics. The positive income tax system additionally includes non-taxable personal income, so that the income total corresponds to household income data information retrieved from the national accounts.

The Korean income tax system is built upon the positive income tax system, so that we estimate total income using personal income information in the national accounts. There are three types of personal income in the accounts that need to be adjusted: imputed employers' social contributions that are the part of the compensation for employees, imputed rents on taxpayers' owned houses that are part of operating surpluses, and indirectly measured financial intermediary services (IMFIS) that are part of property incomes. These three sources of income are not included in tax base although they are attributed to households in the national accounts.

For the exclusion, we follow these procedures. First, employers' social contributions are simply subtracted from employee compensation. The national accounts separately report compensation under three headings: wages, benefits, and social contributions made by employers. The last item is again divided into actual and imputed contributions. The final item is excluded. Second, imputed rents on taxpayers' owner occupied houses are estimated in two steps: we compute the ratio of houses owned by taxpayers using the yearly national census data and multiply the ratio to housing service operating surpluses that are given by the input-output tables each year. The amount of imputed rents on taxpayers' owner occupied houses is estimated by this product. Finally, instead of estimating the amount of IMFIS, we use the information in *the Statistical Yearbook of National Tax* each year that reports household interest and dividend income earned from financial institutions. Using this information for our household property income, there is no need to estimate the IMFIS.

3.3 Population Calculation

Some further remarks on the Korean population data are in order. First, some of the data for those aged 15 or 20 and above involve projections. Statistics Korea conduct a national census every five years which is used to project population over the next five-year period and to correct the prior five-year projections. Currently, the Korean census population data aged 15 or 20 and above for the years 2011 and 2012 are not yet available. We therefore use data projections for these years. Second, the employment and labor force data are based on population bases that are collected monthly by Statistics Korea. The population bases are constructed to include individuals who are capable of working and who are not soldiers, individuals who are required to work in social services (including the police force), and individuals who are incarcerated and serving fixed sentences. Statistics Korea announces the population bases every month and provides detailed statistics segregated by gender, age, and other characteristics. The labor force and employment are estimated by adding to these population bases as required by the definitions of these populations.

4 Supplementary Information for Testing Hypothesis

Table 2 provides the lower and upper bounds (b and u) that were used to test the Pareto distributional hypothesis. Each cell in Table 2 corresponds to each cell in table 4 of Cho, Park, and Phillips (2016). Table 3 provides numerical values of the figures in Figure 1 of Cho, Park, and Phillips (2016).

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Statistics\ Years	2007	2008	2009	2010	2011	2012
Sample Size	10,464,206	11,066,599	11,590178	12,448,203	13,265,840	14,104,742
No. of Groups	2,761	3,352	3,418	3,988	3,553	4,241
First Group	[0.0, 0.5)	[0.0, 0.5)	[0.0, 0.5)	[0.0, 0.5)	[0.0, 0.6)	[0.0, 0.6)
Last Group	[276.40 ∞)	[335.50, ∞)	[342.10, ∞)	[399.10, ∞)	[355.70, ∞)	[424.50, ∞)
Sample Average$_1$	0.3325	0.3361	0.3331	0.3456	0.3582	0.3640
Sample Average$_2$	0.8434	0.8443	0.8443	0.8723	1.0428	1.0334
Sample Variance	2.2668	2.2194	2.1780	2.5191	3.6052	3.4154
Sample Skewness	59.823	67.084	70.287	73.061	59.930	68.368
Sample Kurtosis	6369.6	8075.6	8937.7	9378.1	6498.4	8807.1
Sample Median	138.45	168.00	171.30	199.80	178.10	212.50
Sample Mode Group	[0.5, 0.6)	[0.5, 0.6)	[0.5, 0.6)	[0.5, 0.6)	[0.6, 0.7)	[0.6, 0.7)
Populations ≥ 15	39,873,045	40,459,969	40,949,973	41,434,992	42,008,528	42,445,378
Populations ≥ 20	36,640,987	37,133,082	37,536,274	37,967,813	38,540,049	39,021,687
Labor Forces	39,170,000	39,598,000	40,092,000	40,590,000	41,052,000	41,582,000
Employments	23,433,000	23,577,000	23,506,000	23,829,000	24,244,000	24,681,000

Table 1: Descriptive Statistics of Income Tabulations and Populations of Korea (2007–2012). All groups from the second to the second to last have the group interval size: KRW10 mil. All income statistics are measured in KRW100 mil. Sample Average$_1$ is the average of sample incomes of all individual observations. The other income statistics are obtained by using the group median values and their frequencies from the second to the second to last groups.

Years	Top $x\%$	Statistics \ Populations	$\geq$ 15 year old	$\geq$ 20 year old	measured by Labor Forces	measured by Employments
2007	1.00%	b	0.50	0.50	0.50	0.50
		u	2.50	2.80	2.50	2.50
	0.10%	b	2.10	2.10	2.10	2.10
		u	4.10	4.10	4.10	4.10
	0.05%	b	2.10	2.10	2.10	3.50
		u	4.10	4.10	4.10	6.10
	0.01%	b	8.50	8.50	8.50	12.0
		u	10.50	11.50	10.50	14.00
2008	1.00%	b	0.50	0.50	0.50	0.50
		u	2.50	2.50	2.50	2.50
	0.10%	b	2.40	2.40	2.40	2.40
		u	4.40	4.40	4.40	4.40
	0.05%	b	2.40	2.40	2.40	4.30
		u	4.40	4.40	4.40	6.50
	0.01%	b	9.00	9.00	9.00	12.20
		u	11.00	11.00	11.00	14.20
2009	1.00%	b	0.50	0.50	0.50	0.50
		u	2.50	2.50	2.50	2.50
	0.10%	b	2.40	2.50	2.50	2.50
		u	4.40	5.80	5.80	4.80
	0.05%	b	2.50	2.50	2.50	2.50
		u	5.80	5.80	5.80	5.80
	0.01%	b	8.40	8.40	8.40	12.0
		u	11.00	11.00	11.0	14.20
2010	1.00%	b	0.50	0.50	0.50	0.50
		u	2.50	2.50	2.50	2.50
	0.10%	b	2.20	2.20	2.20	2.20
		u	6.10	6.10	6.10	6.10
	0.05%	b	2.20	2.20	2.20	2.20
		u	6.10	6.10	6.10	6.10
	0.01%	b	8.00	10.00	9.00	10.00
		u	12.00	15.00	12.00	15.00
2011	1.00%	b	0.60	0.60	0.60	0.60
		u	2.60	2.60	2.60	2.60
	0.10%	b	2.80	2.80	2.80	2.80
		u	6.90	6.90	6.90	6.90
	0.05%	b	2.80	2.80	2.80	2.80
		u	6.90	6.90	6.90	6.90
	0.01%	b	10.00	11.00	10.00	15.00
		u	13.00	13.10	13.00	17.00
2012	1.00%	b	0.60	0.60	0.60	0.60
		u	2.60	2.60	2.60	2.60
	0.10%	b	2.00	2.00	2.00	2.00
		u	7.00	7.00	7.00	7.00
	0.05%	b	2.00	2.00	2.00	2.00
		u	7.00	7.00	7.00	7.00
	0.01%	b	9.70	10.50	11.00	15.70
		u	12.00	13.50	13.00	17.90

Table 2: Lower and Upper Bounds for Top Income Estimation of Korea (2007–2012). Notes: b and u are the lower and upper border values of the grouped data. The units of b and u are KRW100 mil.

Top $x\%$	Years \ Populations	$\geq$ 15 year old	$\geq$ 20 year old	measured by Labor Forces	measured by Employments
1.00%	2007	11.45 (11.70)	11.06 (11.19)	11.33 (11.59)	8.63 (8.93)
	2008	11.37 (11.79)	10.80 (11.26)	11.23 (11.65)	8.79 (8.94)
	2009	11.21 (11.69)	10.64 (11.17)	11.07 (11.56)	8.24 (8.82)
	2010	12.25 (12.38)	11.09 (11.83)	11.56 (12.24)	8.57 (9.38)
	2011	12.55 (12.89)	12.07 (12.33)	12.37 (12.74)	9.53 (9.84)
	2012	12.22 (12.29)	11.37 (11.77)	11.82 (12.16)	8.86 (9.40)
0.10%	2007	4.10 (4.10)	3.96 (3.96)	4.07 (4.07)	3.32 (3.31)
	2008	4.10 (4.11)	3.96 (3.96)	4.07 (4.02)	3.32 (3.30)
	2009	4.05 (4.05)	3.90 (3.91)	4.01 (4.02)	3.22 (3.22)
	2010	4.34 (4.34)	4.19 (4.19)	4.30 (4.31)	3.46 (3.46)
	2011	4.50 (4.61)	4.33 (4.45)	4.45 (4.57)	3.56 (3.67)
	2012	4.31 (4.32)	4.16 (4.17)	4.27 (4.28)	3.44 (3.44)
0.05%	2007	3.11 (3.11)	3.01 (3.00)	3.09 (3.08)	2.51 (2.51)
	2008	3.10 (3.10)	3.00 (3.00)	3.07 (3.07)	2.49 (2.49)
	2009	3.04 (3.04)	2.93 (2.93)	3.01 (3.01)	2.42 (2.42)
	2010	3.28 (3.27)	3.17 (3.16)	3.25 (3.25)	2.62 (2.62)
	2011	3.35 (3.45)	3.23 (3.33)	3.32 (3.42)	2.64 (2.75)
	2012	3.24 (3.23)	3.13 (3.12)	3.21 (3.20)	2.58 (2.57)
0.01%	2007	1.61 (1.62)	1.56 (1.56)	1.60 (1.60)	1.28 (1.29)
	2008	1.61 (1.61)	1.56 (1.56)	1.60 (1.60)	1.29 (1.29)
	2009	1.57 (1.57)	1.52 (1.52)	1.56 (1.56)	1.26 (1.26)
	2010	1.72 (1.72)	1.66 (1.66)	1.71 (1.71)	1.38 (1.38)
	2011	1.65 (1.76)	1.59 (1.70)	1.63 (1.74)	1.29 (1.40)
	2012	1.64 (1.64)	1.58 (1.59)	1.63 (1.63)	1.31 (1.31)

Table 3: TOP INCOME SHARES OF KOREA(2007–2012, IN %). The figures show the share of the top $x\%$ income out of total income of each year. The figures are the same shares measured by Atkinson's (2005) mean-split histogram method computed by Park and Jeon (2014).